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MISFOCUSED SPECKLE SHEARING MOIRÉ

BY

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DEDICATION

The research that led to this report
was initially begun by LTC Charles M. Radler,
Associate Professor, Department of Mechanics,
U.S.M.A. This paper is dedicated to his
memory and the spirit of learning that he
so exemplified.

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ABSTRACT

The development of speckle photography is briefly discussed. The technique and theory of misfocused speckle shearing Moiré is examined. A single laser beam is used to illuminate a specimen which is then photographed by a misfocused camera through two laterally separated apertures. The optical phase shift of the recorded speckle grid and the mechanical shift of the speckle is discussed, and an explanation of the combination of the two effects which yields hybrid displacement and strain fringes is presented. Photographs of the fringes on the specimen are compared with plots for the theoretical fringes for four examples to substantiate the theory.

DISCUSSION AND THEORY

The development of the laser, a source of monochromatic, coherent light, spawned a rapid development in the use of photography in the Experimental Mechanics field. The amazing three-dimensional photographs called holograms enabled engineers to experimentally determine displacements of an object by optically comparing the three-dimensional images of an undeformed and deformed specimen. This new field called holographic interferometry however brought with it many problems. The technique was extremely sensitive to any extraneous motion of the experimental setup. The sensitivity was at times far too great for practical use. Also a phenomenon called "speckle" made the images less distinct than desired.

The speckled images were comprised of tiny blotches of light caused by random interference of the reflected light from the surface of the object. These speckles caused the images to lack distinct resolution. Some of the early research done on the speckle phenomenon was with the purpose of eliminating the speckle from the holograms. It wasn't until the late 1960's and early 1970's that the speckle effect was put to constructive use. A very brief explanation of each of the early techniques is presented here.

J. A. Leendertz in 1970 published an article entitled "Interferometric displacement measurements on scattering surfaces utilizing speckle effect." His most useful technique required the specimen to be illuminated simultaneously by two expanded laser beams and photographed by a standard camera. Each beam created a speckle pattern, and these two patterns combined to form a speckled image of the specimen on the film plane. The specimen was then deformed, and the photographic plate was also given a small translation

in its own plane. A second photo of the specimen was then double exposed on the original image. The photographic plate had recorded on it both an undisturbed speckle pattern of the object and a disturbed pattern. When a laser beam was shined through the plate the transmitted light was diffracted by the tiny speckles. The diffraction of light by the speckles plays a major role in all speckle techniques.

The technique of focusing diffracted light onto a plane, filtering only part of the light, and imaging the selected light is called Fourier filtering and is well established. An excellent discussion of the theory and technique can be found in reference 2. A schematic of the Fourier filtering arrangement is shown in Figure 1 on the next page. The filtering is accomplished in the transform plane using an opaque plate with a small hole to allow only the desired light to pass to the camera.

The small displacement of the photographic plate between exposures caused the two superimposed speckle patterns to diffract the light into separate bands when the transmitted light was focused onto the Fourier transform plane. However, the motion of the second speckle pattern due to the deformation of the object also caused some additional diffraction of light. Some of the diffracted light fell on the dark bands of the diffraction pattern in the transform plane. This light carried information only about the deformation of the surface. When just this light was allowed to pass through the transform plane and imaged, fringes representing loci of equal in-plane displacements were observed.

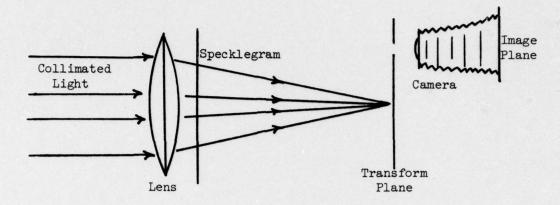


Figure 1 - Fourier Filter

The sensitivity of Leendertz's method is approximately the same as holographic interferometry. That degree of sensitivity is often too great for applications work, and also the necessary translation of the photographic plate is difficult to properly achieve.

In 1970 and 1972 E. Archbold, A. E. Ennos and J. M. Burch described a simpler technique for using speckle to measure larger displacements than the Leendertz procedure. Their method called for illuminating the object with a single laser beam and doubly exposing the film plate before and after deformation. No displacement of the photographic plate is required, but the motion of the surface of the specimen must be greater than the size of the speckle. The article published in 1972 discussed a number of applications and limitations. Rigid body translation, line of sight displacement, rigid body rotation, tilt and surface vibration are all briefly discussed. The theory of the technique was lightly covered in the 1972 article, but the strength of the procedure was obvious. Donald E. Duffy in 1974 paper explained the fringe formation theory of the one beam speckle technique very well, and for completeness in this report the Duffy explanation is summarized here.

If I_1 (p, q) represents the intensity of the speckles recorded on the image plane of the camera before deformation and I_2 (p, q) is the intensity distribution after displacement, then if s is the displacement of the image in the p direction the second recording I_2 (p, q) equals the first recording modified due to the s displacement I_1 (p+s, q). If the negative is developed linearly, the transmission function of the negative is

$$T(p, q) = a - b [I_1 (p, q) + I_1 (p+s, q)]$$
 (1)

where a and b are constants of the photographic film.

Now if a laser beam is passed through the negative and imaged by a lens onto the Fourier transform plane the complex amplitude in the plane is

$$A(u, v) = \exp \left[ik \left(\frac{u^2 + v^2}{2z}\right)\right].$$

$$\iint T(p, q) \exp \left[ik \left(\frac{pu + qv}{z}\right)\right] dp dq \qquad (2)$$

which is the Fourier transform of the transmittance T(p, q) multiplied by a quadratic phase factor. This is the Kirchoff Diffraction formula.²

Substituting the transmittance expression (Eqn. 1) into the complex amplitude expression and then dealing with only those terms that will diffract light outside of the center bright spot (the b terms), the complex amplitude of importance will be

$$A(u, v) = b\{\exp\left[ik \frac{u^2 + v^2}{2z}\right]\}\{1 + \exp\left[-\frac{iKus}{z}\right]\}$$

$$\iint I_1(p, q) \exp\left[-\frac{iK}{z}(up + vq)\right] dp dq \qquad (3)$$

The intensity in the transform plane is

I
$$(u, v) = [A (u, v)]^2$$

Squaring Eqn. 3 and simplifying greatly yields

I (u, v) =
$$4b^2 \left[\cos \frac{Kus}{\lambda z}\right]^2 \cdot I_1$$
 (u, v)

The intensity distribution in the transform plane is the product of the intensity distribution that would be produced by either input image alone modified by a cosine-squared function. The cosine function modulates this intensity to produce a set of equispaced and parallel interference fringes along the u axis.⁵

It can be shown that the separation distance of the fringes is a function of the uniform surface displacement s. If other than a uniform displacement occurs on the surface of the specimen the light diffracted onto the transform plane will be composed of many many sets of fringes. These fringes will each have a different spacing so that now a halo of light will be seen around the central bright spot. By locating a pinhole at any point in the halo (Fourier filtering) a certain sensitivity and component direction can be selected, and the imaged, transmitted light will yield fringes of loci of equal displacements. A photograph of this type of fringe formation is well shown in the article.⁵

Duffy, also in the same article, discussed a modification to the one beam speckle technique. The technique was used in this research. The procedure calls for limiting the spatial frequency content of the image by photographing the illuminated object before and after deformation through two laterally separated aperatures. Each aperature alone would record the speckle pattern, but as the light from the two aperatures

meet on the film plane interference takes place causing interference fringes in each speckle. Thus a periodic grid structure is formed within each speckle of the image. Figure 2 shows a photograph of the speckle and the grid.



Figure 2

These grid lines can be viewed as a moiré grid. When the second exposure of the deformed object is recorded over the first exposure, small local displacements will produce moiré fringes which are contours of equal in-plane displacements. Moiré theory is available in almost any Experimental Mechanics text. The contrast of these fringes can be greatly improved by the Fourier filtering technique discussed earlier. When the diffracted light is focused on the Fourier transform plane it is concentrated at particular points as dictated by the arrangement of the two aperatures. Thus the sensitivity is fixed by the separation of the aperatures and not by the Fourier filtering parameters.

The displacement fringes that can be obtained by the Duffy technique are excellent, however, the engineer is far more interested in strain than in displacement. Graphical techniques are approximate at best for converting displacement data to strain data to be used for stress analysis. A new speckle technique that would yield full field strain fringes would greatly aid experimental engineers.

Y. Y. Hung and C. E. Taylor of the University of Illinois presented a paper in 1973 that described a technique that was "a tool for measurement of derivatives of surface displacement". For this procedure two small glass pieces were placed in front of the Duffy-type double aperatures to shift the two speckle patterns with respect to each other causing a shearing of the images. The authors suggested that the fringe patterns formed were due to the derivatives of displacements which are in some cases strain. A diagram of Hung's setup follows.

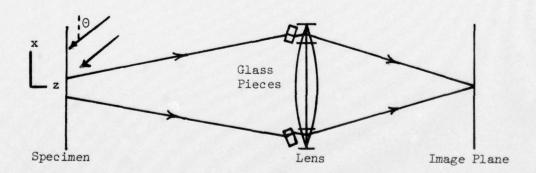


Figure 3

The paper stated that fringes were formed when the following relationship was met.

$$(1 + \sin \theta) \frac{\partial w}{\partial x} + \cos \theta \frac{\partial u}{\partial x} = \frac{n\lambda}{\delta}$$
 (4)

 δ was the amount of shift caused by the glass pieces. The $\frac{\partial w}{\partial x}$ and $\frac{\partial u}{\partial x}$ terms can easily be related to strain quantities. In fact $\frac{\partial \mathbf{u}}{\partial x}$ is the strain in the x direction. The significance of Eqn. 4 will be discussed later as the misfocused speckle moiré theory is developed.

Hung's paper and personal communications with him prompted the investigation by this researcher of the possibility of misfocusing the camera to achieve the shearing effect rather than using glass pieces in front of the aperatures. The physical arrangement is as follows

FRONT FOCUS

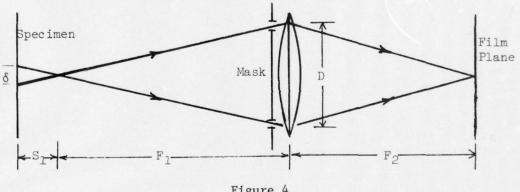


Figure 4

The figure shows a simple schematic of a camera. The plate in front of the lens is the aperature mask containing the Duffy-type aperatures. In general, four aperatures are actually in the mask oriented as shown in figure 5.

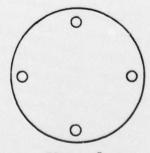


Figure 5

For simplicity only two will be theoretically handled here, but actually each pair forms a corresponding grid in the speckle.

The focal distances F_1 and F_2 are established by the basic relationship

$$\frac{1}{F_L} = \frac{1}{F_1} + \frac{1}{F_2} \tag{5}$$

where $F_{\rm L}$ is the focal length of the camera lens. The object to be photographed is placed an additional distance S_1 away from the front focal point of the camera. This causes the two images formed by the double aperatures to be "sheared" with respect to one another.

Another conceptual approach that facilitates theoretical discussion is that the light rays from two small areas a distance δ apart on object are brought together on the film plane and caused to interfere yielding the same type grid structure as in the Duffy two aperature procedure. The actual nature of speckle and the accurate explanation as to how the extremely small aperatures collect the light from the surface are critical to the theoretical explanation of the technique.

Goldfischer fully explains the speckle formation phenomenon in a 1964 article. For the purposes of this technique the speckles are considered to be made up of the vector sum of the contributions of all significantly close scattering points. Each speckle is assumed to be associated with its particular area of the specimen. Each speckle will move with the surface as it deforms yet will also be altered as the random phases of the contributing scattering points are changed due to straining. The separation distance δ of small speckle producing area is large compared to the speckle diameter.

Another possible physical setup is as follows

"BACK FOCUS"

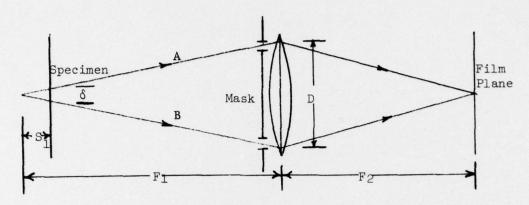


Figure 6

Once again the focal distances are related by equation 5. However, now the object is placed a distance S_1 closer to the lens than the front focal point. Once again one image is shifted with respect to the other or, the two small areas δ apart are brought to interfere on the film plane causing the speckle grid on the misfocused image.

The key question now becomes; how does the motion of the small areas on the object due to deformation carry over to the motion of the grid on the film plane? An examination of some of the parameters will be necessary first. What is the pitch of the recorded grid on the film plane? The following diagram on the next page can be used to determine the separation distance between the grid lines as seen in figure 2. The lines intersecting the film plane represent the wavefronts of rays A and B. When the two wavefronts match up, the film will record the dark grid as denoted by the wide dashes in the figure.

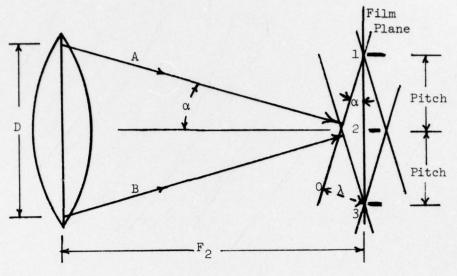


Figure 7

If α is the half-angle formed by the rays \boldsymbol{A} and \boldsymbol{B} from the two aperatures

$$\tan \alpha = \frac{D}{2 F_2}$$
 or for small angles $\alpha = \frac{D}{2 F_2}$

For triangle 1-0-3 the pitch of the interference grid is determined by

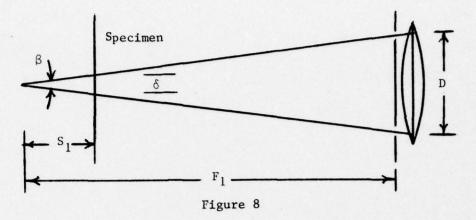
$$\frac{\text{SIDE } 0-3}{\text{SIDE } 1-3} = \frac{\lambda}{2 \text{ (pitch)}} = \sin \alpha$$

Once again for small angles:
$$\sin \alpha = \alpha = \frac{D}{2 \text{ F2}}$$

Therefore $\frac{\lambda}{2 \text{ (pitch)}} = \frac{D}{2 \text{ F2}}$ Yielding: Pitch = $\frac{\lambda}{D}$ (6)

Thus the pitch of the grid is a function only of the internal distances of the camera.

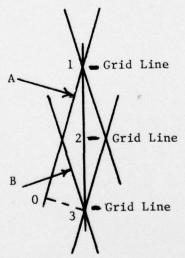
What determines the distance δ between the two small areas on the specimen? Looking at the diagram on the following page for the back focused case the distance δ is easily determined. The front focus case is similar.



we have

angle
$$\beta = \frac{D}{F_1}$$
 $\delta = \beta S_1$ and $\delta = \frac{DS_1}{F_1}$ (7)

Back to the basic question. What causes the grid to shift? Looking at figure 7 it is easily seen that if the wavefronts from ray A were advanced some distance Δ (or in phase by Δ $\frac{2\pi}{\lambda}$) then the location of interference grid points 1, 2, 3 would be shifted downward. This is best shown by a close up of triangle 1-0-3.



A Shift

Shift

Figure 9

The derivation follows:

$$\frac{\Delta}{2 \text{ (shift)}} = \sin \alpha = \alpha = \frac{D}{2 \text{ F}_2}$$

which yields

$$shift = \frac{\Delta F_2}{D}$$
 (8)

In other words if there were a <u>relative phase change</u> $\frac{2\pi}{\lambda}$ between the two sets of rays, the grid will shift a distance $\frac{\Delta}{D}$.

The two sets of rays originate at two separate source areas on the objects surface, and because of the misfocusing they are brought to interfere on the film plane. The motion of the objects surface will cause the relative phase changes between rays reflected from two separate small areas as follows:

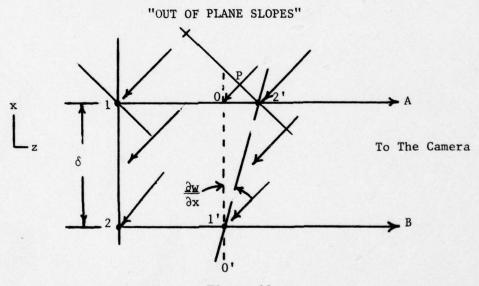


Figure 10

Areas 1 and 2 on the surface of the specimen displace in this case to locations 1' and 2'. If the object had just moved to plane 0-0' no relative phase shift would have resulted between rays A and B. However the forward rotation to the slope $\frac{\partial w}{\partial x}$ does cause a relative phase shift. Looking at the triangle P-0-2'

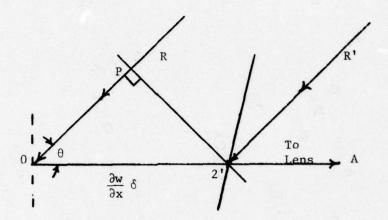


Figure 11

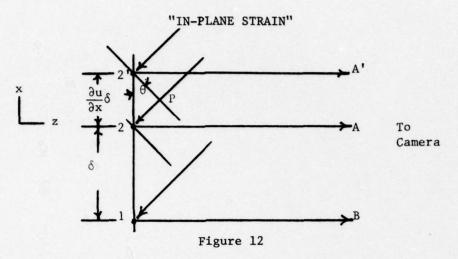
We see that rays R' for the second exposure must travel a shorter distance than rays R for the first exposure. The shortened optical path length will cause the phase of the reflected rays from small area 2' to be advanced with respect to the rays from area 1'.

The distance involved is P \Rightarrow 0 \Rightarrow 2'. From figure 10 we see that distance 0 \Rightarrow 2' is equal to $\frac{\partial w}{\partial x}$ δ . Distance 0 \Rightarrow P then is $\frac{\partial w}{\partial x}$ δ cos θ from

figure 11, which makes the total distance (1 + cos θ) $\frac{\partial w}{\partial x}$ δ . The relative phase advance then is

$$\frac{2\pi}{\lambda}$$
 (1 + cos θ) $\frac{\partial w}{\partial x}$ δ

In-plane strain motion also causes relative phase changes as shown in a very similar derivation on the next page.



The distance between small areas 1 and 2 on the object is increased due to in-plane straining. The extra distance between small areas 1 and 2 is equal to $\frac{\partial u}{\partial x} \delta$, and correspondingly the shorter distance involved in the optical path length between rays A and rays A' is the distance P\Delta 2 or $\frac{\partial u}{\partial x} \delta$ sin θ giving a relative phase advance of rays A' of

$$\frac{2\pi}{\lambda} \frac{\partial \mathbf{u}}{\partial \mathbf{x}} \delta \sin \theta$$

For the direction of illumination and observation in the xz plane, strain in the y direction has no affect on optical path length and therefore induces no relative phase differences. Hence the total expression for relative phase change is

$$\Delta = \frac{2\pi}{\lambda} \delta \left[(1 + \cos \theta) \frac{\partial \mathbf{w}}{\partial \mathbf{x}} + \sin \theta \frac{\partial \mathbf{w}}{\partial \mathbf{x}} \right]$$

This expression is the same relationship derived by Hung for the glass shearing camera paper.⁶ For the misfocused camera the grid motion shift caused by this relative phase shift will be, from equations 7 and 8

$$shift = \frac{S_1 F_2}{F_1} \left[(1 + \cos \theta) \frac{\partial w}{\partial x} + \sin \theta \frac{\partial u}{\partial x} \right] = \frac{n\lambda F_2}{D}$$
 (9)

If the other pair of aperatures was considered, equation 9 would contain the partials with respect to y. If the illumination were in the y-z plane the $\frac{\partial u}{\partial x}$ term would be $\frac{\partial v}{\partial x}$.

A very important contribution to grid motion in shearing cameras has previously been overlooked in the research. Any current theory must consider the fact that the source pair of small areas on the object are also relocated as a pair due to the deformation of the specimen.

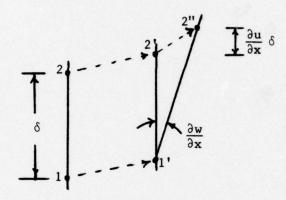


Figure 13

The straining and tilting of the two small areas result in <u>optical</u> phase difference grid shifts as just discussed, but also the "mechanical" shift of areas 1 and 2 to 1' and 2' will cause the interference on the film plane to occur <u>at a different point!</u> This mechanical shift is exactly that which causes the Duffy displacement fringes described in the beginning of this report.

Intuition suggests that the mechanical shift of the grid due to the source pair displacement combines or couples with the optical phase shift of the grid to form hybrid fringes that are complex combinations of displacement and strain fringes. The most simple way of viewing misfocused speckle is in terms of classical moiré fringe formation. Grid motion of one-half the pitch will yield a dark fringe. How do the optical and mechanical effects cause the moiré grid motion?

In basic photography the magnification of the photograph is determined from the ratio of the internal focal length to the front focal length $\frac{F_2}{F_1}$. Length measurements and hence displacements are multiplied by this factor. Therefore the mechanical shift due to source pair displacement is magnified according to the $\frac{F_2}{F_1}$ ratio. That is, the magnification factor affects the mechanical grid motion just as it magnifies dimensions. For a magnification ratio of 1.5 a unit displacement on the object will be recorded as a 1.5 unit displacement on the film plane.

The misfocusing however complicates the ratio. If the arrangement is as shown in figure 6, the back focus case, one should note that the object is not located at the front focal point of the lens. Therefore the misfocused distance S_1 , must be considered in the ratio. It is easily shown by similar triangles that the new ratio is $\frac{F_2}{F_1 - S_1}$ for

the back focus set-up. For the front focus case, figure 4, the ratio is $\frac{F_2}{F_1+S_1}$, and any mechanical source pair displacements will be magnified

by that factor when recorded on the film plane.

The magnification ratio has no effect on the optical (relative phase difference) shift of the recorded grid. Once the reflected ray groups leave the surface of the object their relative phase difference is fixed. No matter how far the rays travel they will maintain the same relative phase relationship and will correspondingly interfere in the same manner on the film plane.

However, for each of the two set-ups discussed, one where the front focal point is behind the specimen and the other where the focal point is in front of the specimen, the combination of the mechanical and optical effects are different. Looking at the instance where the focal point is behind the specimen (back focus)

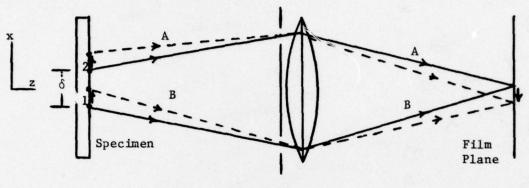
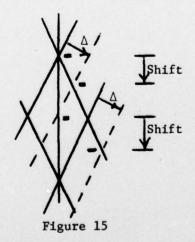


Figure 14

We see that a mutual displacement of the pair of small areas 1 and 2 in the positive x direction on the specimen will move the interfering speckle in the negative x direction on the film plane due to the image inversion of the camera. Now if positive strain occurs between areas 1 and 2, that is if 1 remains stationary, and 2 moves in the positive x direction, light ray group A will experience an advance in phase $(\Delta \frac{2\pi}{\lambda})$ relative to rays B. This advance in phase will shift the grid in the negative x direction on the film plane as diagrammed below.



Thus the positive displacement effects and the positive straining effects are additive. Since the image is inverted on the film plane the relationship of grid motion to object image is correct. Now similarly the other cases of straining follow logically, and hence the grid motion on the film plane is the sum of the mechanical and optical effects

$$\text{or } \frac{F_2}{F_1 \, - \, S_1} \, \, u \, + \, \left[(1 \, + \, \cos \, \theta) \, \, \frac{\partial w}{\partial x} \, + \, \sin \, \theta \, \, \frac{\partial u}{\partial x} \right] \, \frac{S_1 F_2}{F_1}$$

This motion is reflected in grid motion which has a pitch or recording sensitivity of $\frac{n\lambda F_2}{D}$.

If the focal point of the camera lens is in front of the specimen (front focus) a different combination develops.

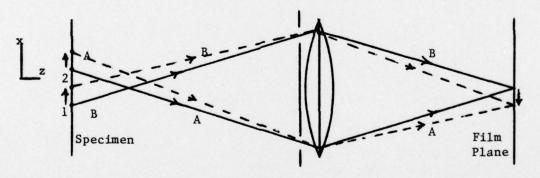


Figure 16

As before if the two areas displace together in the positive x direction the film plane will record a mechanical shift in the negative x direction. Note however that in this situation the light ray groups are shown as crossing. Thus, if as described in the back focus case a positive strain occurs, the light rays emanating from small area 2 will undergo a relative advance in phase $(\frac{\Delta 2\pi}{\lambda})$ with respect to ray group B. This phase advance however will have the opposite effect as before since the rays are crossing.

The advanced rays A approach the film plane from the other aperature, and the phase advance will move the recorded grid in the positive x direction or the film plane. Similarly the other cases of strain and displacement follow logically. Thus the displacement and strain effects are <u>subtractive</u>, and the grid motion is the difference of the mechanical and optical effects or

$$\frac{F_2}{F_1 + S_1} u - \left[(1 + \cos \theta) \frac{\partial w}{\partial x} + \sin \theta \frac{\partial u}{\partial x} \right] \frac{S_1 F_2}{F_1}$$

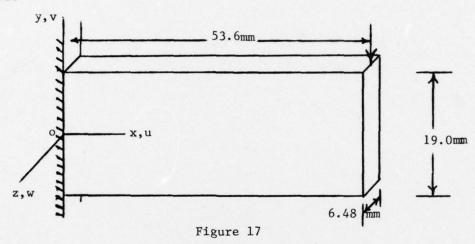
The general expression therefore for the misfocused speckle shearing moiré theory is summarized as:

$$\frac{F_2}{F_1 - S_1} u + \frac{S_1 F_2}{F_1} \left[(1 + \cos \theta) \frac{\partial w}{\partial x} + \sin \theta \frac{\partial u}{\partial x} \right] = \frac{n \lambda F_2}{D}$$
 (10)

where the sign for the misfocused distance S_1 is positive for the back focused case and negative for the front focus set up.

PHOTOGRAPHIC RESULTS

The familiar cantilever beam was used as the experimental vehicle for the misfocused speckle moiré technique. The displacement fields are well defined, and rigid body motion is prevented. The arrangement, coordinate system and dimensions for the plexiglass beam were as shown below.



Loading was accomplished using a micrometer mounted in a rigid support thus giving a controlled end displacement. The illumination was in the x-z plane, and the angle θ formed with the surface normal was 30° . With this arrangement the theoretical expression derived in the theory section is valid. That is the derivatives $\frac{\partial u}{\partial x}$ and $\frac{\partial w}{\partial x}$ contribute to the optical shift of the grid, and u displacements yield a mechanical shift.

The camera used for this work was the Photoelastic, Inc., moiré camera model 103 with a Nikon lens having a focal length of 305mm. The aperature mask was the same as shown in figure 5. This was placed in front of the lens like a lens cap. The additional separation distance of the optical aperatures due to the forward location of the mask was compensated for.

The actual separation distance, D, was 26.4mm. Due to the extremely low light level reaching the film plane, SO 253, a very fast KODAK holographic film, was used. Exposure times were approximately 8 minutes for each exposure. The laser was a 15 milliwatt He-Ne, and the beam was expanded and collimated just large enough to fully illuminate the specimen. The beam was spray painted white to increase reflectivity.

Since the theory predicts the resultant fringes to be hybrid combinations of displacement and strain contributions a look at computer simulated fringes is presented for comparison. In the first case an end displacement of the cantilever beam of .153mm theoretically yields a displacement field fringe pattern as shown in figure 18. The corresponding $\frac{\partial u}{\partial x}$ strain fringes for that displacement field are shown in figure 19. The $\frac{\partial w}{\partial x}$ contribution is so small that the fringes do not plot at this sensitivity. The equations for the displacement field of an end loaded cantilever beam are available in elasticity texts. 8

Looking first at how the back focus (additive) arrangement combines the displacement and strain contributions, the theoretical expression from equation 10 is

 $1.05\text{u} + 30.48 \left[(1 + \cos 30^\circ) \, \frac{\partial \text{w}}{\partial \text{x}} + \sin 30^\circ \, \frac{\partial \text{u}}{\partial \text{x}} \, \right] = .0146\text{n} \quad (11)$ The misfocused distance S_1 for this example was +30.48mm. The focal distances were $F_1 = 610\text{mm}$ and $F_2 = 610\text{mm}$, and the illumination angle θ was 30° . The plot of equation 11 is shown in figure 20, and the actual photograph for the back focus arrangement is shown in figure 21. The results are obviously very close to the theoretical plot. The photograph does show a slight neutral axis shift due probably to a lack of full

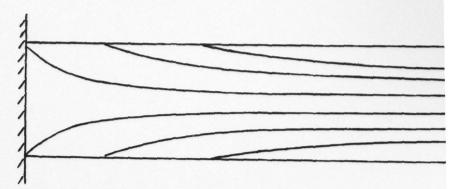


Figure 18

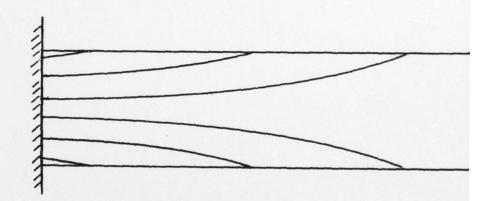


Figure 19

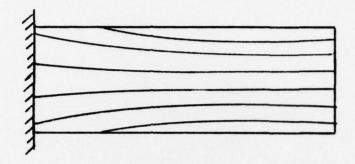


Figure 20

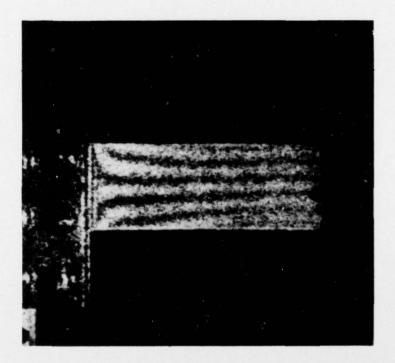


Figure 21

rigidity in the fixed support.

Now with the <u>exact same</u> displacement of the end of the cantilever beam but with the camera set up in the front focus (subtractive) arrangement, the theoretical plot of expected fringes is as shown in figure 22. The mathematical expression used for the plot of the combination of effects is shown below.

.95u - 30.48 [(1 + cos 30°) $\frac{\partial w}{\partial x}$ + sin 30° $\frac{\partial u}{\partial x}$] = .0146n (12) The only difference between equations 11 and 12 is the sign of the misfocused distance S1. For the front focus arrangement S1 was -30.48. The actual front focused photograph is figure 23. The plot and the photograph compare very closely. Note that the only difference between the photographs was the misfocused distance S1. The camera was merely moved forward or backward on its support. None of the internal optical distances were changed, the loading was exactly the same, the illumination angle was exactly the same, and the Fourier filtering method was precisely the same. In the first case, the focal plane was 30.48mm behind the specimen, and for the second it was 30.48mm in front of the object.

A second set of the many results is shown in the next figures. The optical dimensions are considerably changed, and the end displacement was increased. The misfocused distances were greater ($S_1 = \pm 36.83$) hence the photographs are less distinct. The back focus case is shown in figures 24 and 25 and the front focus is shown in figures 26 and 27. The optical distances F_1 and F_2 were 582.3 and 640.5 respectively and θ remained 30°. Once again the shape and location of the fringes match very well, and the results substantiate the theory.

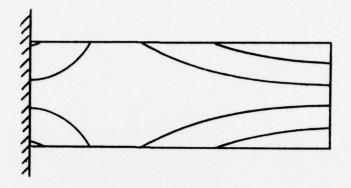


Figure 22

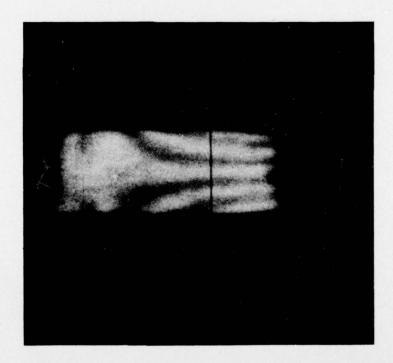


Figure 23

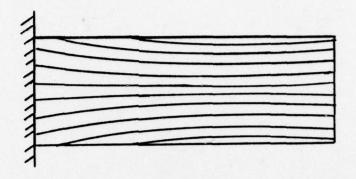


Figure 24

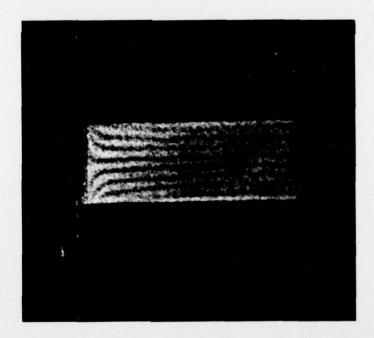


Figure 25

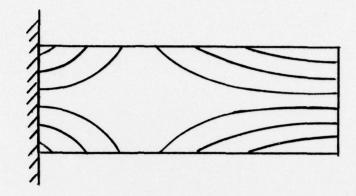


Figure 26



Figure 27

NUMERICAL RESULTS

The optical contributions can be separated from the mechanical contributions mathematically in the data reduction process. Taking equation 11 and equation 12 for the case presented first in the photographic results and subtracting out the "u" terms. (The bracket represents the strain terms.)

$$.95u - 30.48$$
 [] = $.0146n_{FF}$
-(1.05u + 30.48 [] = $.0146n_{BF}$) x $.905$

The resulting expression

$$(1 + \cos 30^{\circ}) \frac{\partial w}{\partial x} + \sin 30^{\circ} \frac{\partial u}{\partial x} = 2.5 \times 10^{-4} (.905 n_{BF} - n_{FF})$$

can be used to determine the "strain" expression at any point on the specimen. By determining the fringe number for the back focus case $n_{\rm BF}$ and for the front focus case $n_{\rm FF}$ from the photographs and substituting into the above expression, results have been obtained with an accuracy of between 5 and 25 percent. The major source of the error is believed to be due to the neutral axis shift as a result of the non-rigidity of the fixed support.

CONCLUSION

Misfocused speckle shearing photography appears to be a possible method for determining strain data on the surface of a specimen. Further work is necessary to improve fringe visibility and to optically separate each of the $\frac{\partial w}{\partial x}$ and $\frac{\partial u}{\partial x}$ contributions of the strain term.

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